Estimation de Volatilité en Présence de Bruit de Microstructure Endogène

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Résumé: Ce papier considère des procédures statistiques facilement implémentables pour l'estimation de mesures de volatilité haute fréquence pour des actifs financiers. Le modèle de microstructure sous-jascent se base sur un prix efficient de type semi-martingale continue et permet de reproduire les principales caractéristiques empiriques des données ultra haute fréquence. Dans ce modèle, le bruit de microstructure est endogène mais ne dépend pas uniquement du prix efficient. En utilisant les prix de transaction observés, nous développons une nouvelle approche permettant d'approximer les valeurs du prix efficient à certains instants aléatoires. En se basant sur ces valeurs approchées, on construit un estimateur de la volatilité intégrée et on fournit sa théorie asymptotique. On donne aussi un estimateur consistant de la co-volatilité intégrée dans le cas où deux actifs (asynchrones par construction du modèle) sont observés.

Mots clés: Bruit de microstructure; Données ultra haute fréquence; Volatilité; Co-volatilité; Durations; Données asynchrones; Temps d'arrêt; Martingales.

Abstract: This paper considers practically appealing procedures for estimating intraday volatility measures of financial assets. The underlying microstructure model accommodates the inherent properties of ultra high frequency data with the assumption of continuous efficient price processes. In this model, the microstructure noise is endogenous but does not only depend on the prices. Using the (observed) last traded prices of the assets, we develop a new approach that enables to approximate the values of the efficient prices at some random times. Based on these approximated values, we build an estimator of the integrated volatility and give its asymptotic theory. We also give a consistent estimator of the integrated co-volatility when two assets (asynchronous by construction of the model) are observed.

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